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Frictional heating during braking in a three-element tribosystem

A.A. Yevtushenko *, M. Kuciej

Department of Applied Mechanics and Informatics, Faculty of Mechanical Engineering, Technical University of Bialystok, 45C Wiejska Street, Bialystok 15-351, Poland

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ABSTRACT

A three-element model of braking process is proposed. In order to determine the temperature fields in each element of the model, the analytical solution of a boundary-value problem of heat conduction for tribosystem, consisting of the semi-space, sliding with the time-dependent velocity (braking at uniform retardation) on a surface of the strip deposited on a semi-infinite foundation, is obtained. The results of the numerical analysis for different materials applied in a braking system, cast iron–FMK-11 metal ceramics–steel, are presented.

1. Introduction

It is well known that mechanical energy is transformed into thermal energy whenever friction occurs. This frictional heating is responsible for temperature increase of the contact surface of bodies and has considerable influence on the tribological behaviour. Therefore, the problem of frictional heat is of a great theoretical and practical interest to researchers.

The heating problems of friction can be examined through a stationary, quasi-stationary and a nonstationary statements. If the slip velocity is low then the convection caused by the motion does not change the temperature and heat fluxes, as well as the process of heat conduction for given external conditions lasting long enough that the influence of initial conditions can be ignored, then in the previous both cases the thermal contact can be assumed as stationary one. A quasi-stationary thermal contact takes place under the condition of sufficiently long duration of friction between bodies being in motion. Whereas, a nonstationary thermal contact is either conditioned by a nonstationary distribution of the contact pressure or by a time-dependent slip velocity as well as by the fact that the development of heating process is considered from some initial time.

The thermal processes during braking are nonstationary and of short duration. A criterion for evaluation of the frictional thermal strength of materials applied in the contacting pairs, in which the principal role plays the temperature of friction has been proposed by Chichinadze [1]. In the latter paper by Chichinadze et al. [2] the following algorithm for calculation of contact temperature for various types of braking systems was proposed:

– maximum temperature rise in the contact surface is given as the sum of the flash temperature and the average temperature of the nominal contact area (or its contour) caused by a heat flux on its surface;
– for calculation of the temperature flash, sliding of a pin on the surface of a smooth semi-space is considered;
– the average temperature is obtained from a solution of a one-dimensional contact problem with transient frictional heat generation.

Generally speaking, the one-dimensional models correspond to those cases when the heat flux can be assumed as normal to the contact surface (Peclet number must be large). The verification of many analytical solutions with the results from the experimental data which refer to the work of the braking devices, shows that the one-dimensional models may be considered as sufficiently good approximation for the computation of the brake systems with heat generation taken into account [3,4]. The theoretical model for average temperature calculation and wear during braking, is proposed in the papers [5,6]. The model is based on the assumption that the friction elements can be treated as a semi-spaces. The assumption is valid when the operating conditions and frictional heating regime are such that a deep layers of the working elements don't have any considerable influence on the contact temperature. But still exist so called heavy friction modes as for example the aircraft brakes systems, when working elements of brakes are heated through their thicknesses. The solution of the heat problem of

* Corresponding author. Tel.: +48 0857469200; fax: +48 085749240.
E-mail address: ayevt@pb.edu.pl (A.A. Yevtushenko).

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Nomenclature

d	strip thickness
$\mathrm{erf}(x)$	Gauss error function
$\mathrm{erfc}(x) = 1-\mathrm{erf}(x)$	complementary error function
$\mathrm{ierfc}(x) = \pi^{-1/2}\exp(-x^2) - x\,\mathrm{erfc}(x)$	integral of the error function $\mathrm{erfc}(x)$
f	frictional coefficient
$H()$	Heaviside's step function
K	coefficient of heat conduction
k	coefficient of thermal diffusivity
p_0	pressure
$q = fVp_0$	intensity of the frictional heat flux (the friction power)
T	temperature
$T_{\max}$	maximal temperature
$T_0 = qd/K$	temperature scaling factor
$T^* = T/T_0$	dimensionless temperature
t	time
$t_{\max}$	time, when maximal temperature is reached
t_s	braking time
V	velocity sliding
V_0	initial velocity sliding
z	spatial coordinate

Greek symbols

$\tau = k_s t/d^2$	dimensionless time (Fourier's number)
$\tau_s = k_s t_s/d^2$	dimensionless braking time
$\zeta = z/d$	dimensionless coordinate

Indexes

f	bottom semi-space (foundation)
s	strip
t	upper semi-space (top)

friction during braking in the case of the finite thickness of the brakes working elements has been obtained in papers [7,8].

The metal ceramics and mineral-ceramic frictional materials are widespreadily used in brake systems nowadays [9]. This could be explained by their high thermal stability and high wear resistance [10]. A friction patch of a brakes is designed as a thin cermet strip based either on iron or on copper. In the process of braking, this patch is pressed to the counterbody (brake drum, disk, rim of the wheel, etc.). As a result of the friction action on the contact surface, the kinetic energy transforms into heat. The elements of brakes are heated and, hence, the conditions of operation of the friction patches become less favourable: their wear intensifies and the friction coefficient decreases, which may lead to emergency situations [11]. Thus, the problem of heating limitation of is one of the most important in brakes design [12].

In the present paper, we derived the solution of the thermal problem of friction for a tribosystem consisting of three bodies: the upper semi-space (the grey cast iron disk) sliding with the velocity $V(t) = V_0(1-t/t_s)$, $0 \leqslant t \leqslant t_s$ (braking with constant retardation) on a surface of the strip (FMK-11 cermet frictional element of the patch) deposited on a semi-space (the steel foundation of a patch). The corresponding problem at V = const. (the uniform sliding) has been studied in article [13].

2. Problem formulation

The problem of contact interaction of two semi-spaces is considered, where one of them is homogeneous and the other is a semi-infinite foundation with a strip of thickness d deposited on its surface. The constant pressures p_0 in direction of z axis of the Cartesian system of coordinates $Oxyz$ are applied to the infinities in semi-spaces (Fig. 1). The upper semi-space slides with velocity

$$V(t) = V_0\left(1 - \frac{t}{t_s}\right)H(t_s - t), \quad t \geq 0, \tag{1}$$

in the direction of the y axis on the strip surface. Due to friction, the heat is generated on a contact plane $z = 0$. It is supposed, that the

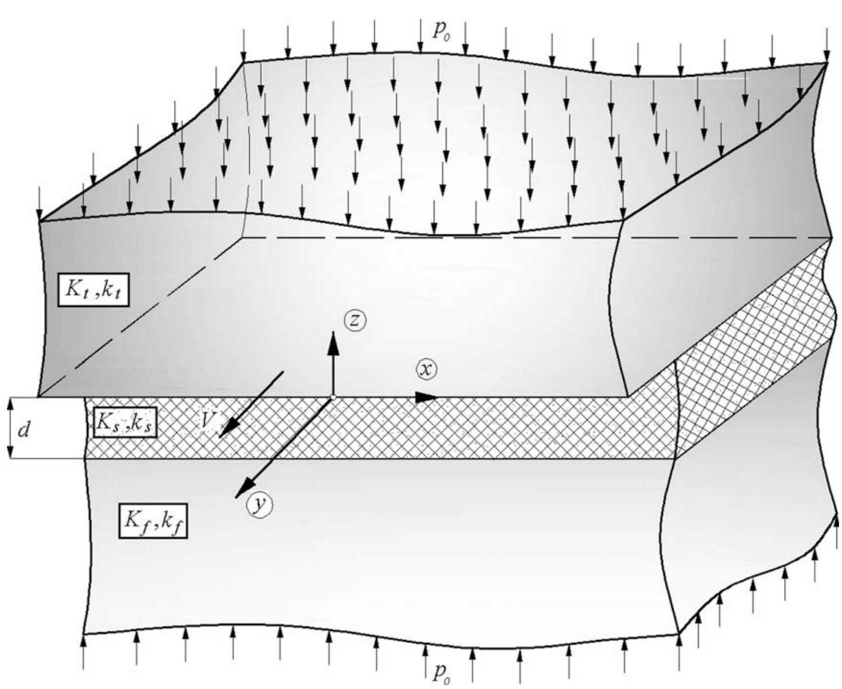

Fig. 1. Scheme of a three-element brake system.

sum of the intensities of frictional heat fluxes directed into each component of a friction pair is equal to the specific friction power [14,15].

Let us find the distribution of temperature fields and intensities of heat fluxes in the frictional elements. Further, all temperatures and the physical parameters concerning a top semi-space, strip and foundation will have bottom indexes "t", "s", and "f", respectively (Fig. 1).

The transient temperature fields $T_{t,s,f}(z,t)$ can be found from the solution of the following transient heat conduction problem of friction:

$$\frac{\partial^2 T_t(z,t)}{\partial z^2} = \frac{1}{k_t}\frac{\partial T_t(z,t)}{\partial t}, \quad 0 < z < \infty,\ t > 0, \tag{2}$$

$$\frac{\partial^2 T_s(z,t)}{\partial z^2} = \frac{1}{k_s}\frac{\partial T_s(z,t)}{\partial t}, \quad -d < z < 0,\ t > 0, \tag{3}$$

$$\frac{\partial^2 T_f(z,t)}{\partial z^2} = \frac{1}{k_f}\frac{\partial T_f(z,t)}{\partial t}, \quad -\infty < z < -d,\ t > 0, \tag{4}$$

$$T_s(0,t) = T_t(0,t), \quad t \geq 0, \tag{5}$$

$$K_s\frac{\partial T_s}{\partial z}\bigg|_{z=0-} - K_t\frac{\partial T_t}{\partial z}\bigg|_{z=0+} = q(t), \quad t \geq 0, \tag{6}$$

$$T_s(-d,t) = T_f(-d,t), \quad t \geq 0, \tag{7}$$

$$K_s\frac{\partial T_s}{\partial z}\bigg|_{z=-d+} = K_f\frac{\partial T_f}{\partial z}\bigg|_{z=-d-}, \quad t \geq 0, \tag{8}$$

$$T_t(z,t) \to 0,\ z \to \infty,\ t \geq 0, \tag{9}$$

$$T_f(z,t) \to 0,\ z \to -\infty,\ t \geq 0, \tag{10}$$

$$T_t(z,0) = 0, \quad 0 \leq z < \infty, \tag{11}$$

$$T_s(z,0) = 0, \quad -d \leq z \leq 0, \tag{12}$$

$$T_f(z,0) = 0, \quad -\infty < z \leq -d, \tag{13}$$

where taking relation (1) into account we have

$$q(t) = q_0 q^*(t), \quad t \geq 0, \tag{14}$$

$$q_0 = fV_0p_0, \quad q^*(t) = \left(1 - \frac{t}{t_s}\right)H(t_s - t),\ t \geq 0. \tag{15}$$

Let us denote by

$$\zeta = \frac{z}{d}, \quad \tau = \frac{k_s t}{d^2}, \quad \tau_s = \frac{k_s t_s}{d^2},$$
$$K_f^* = \frac{K_f}{K_s}, \quad K_t^* = \frac{K_t}{K_s}, \quad k_f^* = \frac{k_f}{k_s}, \quad k_t^* = \frac{k_t}{k_s}, \tag{16}$$

$$T_0 = \frac{q_0 d}{K_s}, \quad T_t^* = \frac{T_t}{T_0}, \quad T_s^* = \frac{T_s}{T_0}, \quad T_f^* = \frac{T_f}{T_0}. \tag{17}$$

By taking denotes (16) and (17) into account, the parabolic boundary-value problem of heat conduction (2)–(13) can be written down in the following form:

$$\frac{\partial^2 T_t^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k_t^*}\frac{\partial T_t^*(\zeta,\tau)}{\partial \tau}, \quad 0 < \zeta < \infty,\ \tau > 0, \tag{18}$$

$$\frac{\partial^2 T_s^*(\zeta,\tau)}{\partial \zeta^2} = \frac{\partial T_s^*(\zeta,\tau)}{\partial \tau}, \quad -1 < \zeta < 0,\ \tau > 0, \tag{19}$$

$$\frac{\partial^2 T_f^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k_f^*}\frac{\partial T_f^*(\zeta,\tau)}{\partial \tau}, \quad -\infty < \zeta < -1,\ \tau > 0, \tag{20}$$

$$T_s^*(0,\tau) = T_t^*(0,\tau), \quad \geq 0, \tag{21}$$

$$\frac{\partial T_s^*}{\partial \zeta}\bigg|_{\zeta=0-} - K_t^*\frac{\partial T_t^*}{\partial \zeta}\bigg|_{\zeta=0+} = q^*(\tau), \quad \tau \geq 0, \tag{22}$$

$$T_s^*(-1,\tau) = T_f^*(-1,\tau), \quad \tau \geq 0, \tag{23}$$

$$\frac{\partial T_s^*}{\partial \zeta}\bigg|_{\zeta=-1+} = K_f^*\frac{\partial T_f^*}{\partial \zeta}\bigg|_{\zeta=-1-}, \quad \tau \geq 0, \tag{24}$$

$$T_t^*(\zeta,\tau) \to 0, \quad \zeta \to \infty,\ \tau \geq 0, \tag{25}$$

$$T_f^*(\zeta,\tau) \to 0, \quad \zeta \to -\infty,\ \tau \geq 0, \tag{26}$$

$$T_t^*(\zeta,0) = 0, \quad 0 \leq \zeta < \infty, \tag{27}$$

$$T_s^*(\zeta,0) = 0, \quad -1 \leq \zeta \leq 0, \tag{28}$$

$$T_f^*(\zeta,0) = 0, \quad -\infty < \zeta \leq -1, \tag{29}$$

where

$$q^*(\tau) = \left(1 - \frac{\tau}{\tau_s}\right)H(\tau_s - \tau), \tau \geq 0. \tag{30}$$

3. Problem solution

We perform the Laplace integral transform [16]

$$\bar{T}_{t,s,f}^*(\zeta,p) = \int_0^\infty T_{t,s,f}^*(\zeta,\tau)\exp(-p\tau)d\tau \tag{31}$$

on the heat conduction Eqs. (18)–(20) and the boundary conditions (21)–(26) with the homogeneous initial conditions (27)–(29) for the temperature. Thus we have

$$\frac{d^2\bar{T}_t^*(\zeta,p)}{d\zeta^2} - \frac{p}{k_t^*}\bar{T}_t^*(\zeta,p) = 0,\ 0 < \zeta < \infty, \tag{32}$$

$$\frac{d^2\bar{T}_s^*(\zeta,p)}{d\zeta^2} - p\bar{T}_s^*(\zeta,p) = 0,\ -1 < \zeta < 0, \tag{33}$$

$$\frac{d^2\bar{T}_f^*(\zeta,p)}{d\zeta^2} - \frac{p}{k_f^*}\bar{T}_f^*(\zeta,p) = 0,\ -\infty < \zeta < -1, \tag{34}$$

$$\bar{T}_s^*(0,p) = \bar{T}_t^*(0,p), \tag{35}$$

$$\frac{d\bar{T}_s^*(\zeta,p)}{d\zeta}\bigg|_{\zeta=0-} - K_t^*\frac{d\bar{T}_s^*(\zeta,p)}{d\zeta}\bigg|_{\zeta=0+} = \bar{q}^*(p) \tag{36}$$

$$\bar{T}_s^*(-1,p) = \bar{T}_f^*(-1,p), \tag{37}$$

$$\frac{d\bar{T}_s^*(\zeta,p)}{d\zeta}\bigg|_{\zeta=-1+} = K_f^*\frac{d\bar{T}_f^*(\zeta,p)}{d\zeta}\bigg|_{\zeta=-1-}, \tag{38}$$

$$\bar{T}_t^*(\zeta,p) \to 0,\ \zeta \to \infty, \tag{39}$$

$$\bar{T}_f^*(\zeta,p) \to 0,\ \zeta \to -\infty. \tag{40}$$

By using technique found in the paper [13] we obtain that the solutions of the ordinary differential Eqs. (32)–(34) which satisfies the boundary condition (35)–(40) have the form:

$$\bar{T}_t^*(\zeta,p) = \frac{\bar{q}^*(p)}{(1+\varepsilon_t)\sqrt{p}}\sum_{n=0}^{\infty}\Lambda^n\left\{\exp\left[-\left(2n + \frac{\zeta}{\sqrt{k_t^*}}\right)\sqrt{p}\right]\right.$$
$$\left. + \lambda_f\exp\left[-\left(2n + 2 + \frac{\zeta}{\sqrt{k_t^*}}\right)\sqrt{p}\right]\right\}, 0 \leq \zeta < \infty, \tag{41}$$

$$\bar{T}_s^*(\zeta,p) = \frac{\bar{q}^*(p)}{(1+\varepsilon_t)\sqrt{p}}\sum_{n=0}^{\infty}\Lambda^n\{\exp[-(2n-\zeta)\sqrt{p}]$$
$$+ \lambda_f\exp[-(2n+2+\zeta)\sqrt{p}]\} - 1 \leq \zeta \leq 0, \tag{42}$$

$$\bar{T}_f^*(\zeta,p) = \frac{2\bar{q}^*(p)}{(1+\varepsilon_t)(1+\varepsilon_f)\sqrt{p}}\sum_{n=0}^{\infty}\Lambda^n\exp\left\{-\left[2n + 1 - \frac{(1+\zeta)}{\sqrt{k_f^*}}\right]\right.$$
$$\left.\sqrt{p}\right\} - \infty < \zeta \leq -1, \tag{43}$$

where

$$\varepsilon_t \equiv \frac{K_t^*}{\sqrt{k_t^*}} = \frac{K_t}{K_s}\sqrt{\frac{k_s}{k_t}}, \quad \varepsilon_f \equiv \frac{K_f^*}{\sqrt{k_f^*}} = \frac{K_f}{K_s}\sqrt{\frac{k_s}{k_f}} \tag{44}$$

$$\Lambda^n = \begin{cases} \lambda^n, & 0 \leq \lambda < 1, \\ (-1)^n|\lambda|^n, & -1 < \lambda \leq 0, \end{cases} \tag{45}$$

$$\lambda = \lambda_t \lambda_f, \quad \lambda_t = \frac{1-\varepsilon_t}{1+\varepsilon_t}, \quad \lambda_f = \frac{1-\varepsilon_f}{1+\varepsilon_f}, \tag{46}$$

$\bar{q}^*(p)$ is the Laplace transform (31) of the heat flux intensity (30).

Inversion of the formulas (41)–(43) is made by the convolution theorem for Laplace's integral transform [16]:

$$L_k(\tau) \equiv L^{-1}[\bar{q}_k^*(p)\bar{Q}(p);\tau] = \int_0^\tau q_k^*(\xi)Q(\tau-\xi)d\xi, \quad \tau > 0, \ k = 0,1, \tag{47}$$

where

$$q_k^*(\tau) = \left(\frac{\tau}{\tau_s}\right)^k, \quad \bar{q}_k^*(p) = \frac{1}{p(p\tau_s)^k}, \quad k = 0,1, \tag{48}$$

$$Q(\tau) = \frac{1}{\sqrt{\pi\tau}}\exp\left(-\frac{a}{4\tau}\right), \quad \bar{Q}(p) = \frac{1}{\sqrt{p}}\exp(-\sqrt{ap}), \quad a > 0. \tag{49}$$

Based on formulas (48) and (49), functions $L_k(\tau)$ (47) are written as:

$$L_k(\tau) = \frac{1}{\sqrt{\pi}}\int_0^\tau \left(\frac{\xi}{\tau_s}\right)^k \frac{\exp\left[-\frac{a}{4(\tau-\xi)}\right]}{\sqrt{\tau-\xi}}d\xi, \quad \tau > 0, \quad k = 0,1. \tag{50}$$

For calculation of integrals in the right-hand side of Eq. (50) we use the substitution

$$u = \frac{a}{4(\tau-\xi)}, \quad \omega = \frac{1}{2}\sqrt{\frac{a}{\tau}}. \tag{51}$$

Then, from (50) it follows that

$$L_0(\tau) = \frac{1}{2}\sqrt{\frac{a}{\pi}}I_0(\omega), \quad L_1(\tau) = \frac{1}{2}\sqrt{\frac{a}{\pi}}\left(\frac{\tau}{\tau_s}\right)[I_0(\omega) - \omega^2 I_1(\omega)], \tag{52}$$

where

$$I_k(\omega) = \int_{\omega^2}^\infty \frac{\exp(-u)}{u^{k+1}\sqrt{u}}du, \quad k = 0,1. \tag{53}$$

Integrating (53) by parts we find [17]:

$$I_0(\omega) = 2\sqrt{\pi}\frac{\mathrm{ierfc}(\omega)}{\omega}, \quad I_1(\omega) = \frac{2}{3}\left[\frac{\exp(-\omega^2)}{\omega^3} - I_0(\omega)\right]. \tag{54}$$

Substituting functions $I_k(\omega)$, $k = 0,1$ (54) into formulas (52) we obtain finally:

$$L_k(\tau) = 2\sqrt{\tau}\left(\frac{\tau}{\tau_s}\right)^k F_k(\omega), \quad k = 0,1, \tag{55}$$

where

$$F_0(\omega) = \mathrm{ierfc}(\omega), \quad F_1(\omega) = 3^{-1}[2(1+\omega^2)\mathrm{ierfc}(\omega) - \omega\,\mathrm{erfc}(\omega)]. \tag{56}$$

Taking into account the form of function $L_k(\omega)$, $k = 0,1$ (55) and (56) from solutions (41)–(43) we find dimensionless temperatures $T^{(k)*}(\zeta,\tau)$, $k = 0,1$ for intensities of heat fluxes $q_k^*(\tau), k = 0,1$ (48):

$$T_t^{(k)*}(\zeta,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\left(\frac{\tau}{\tau_s}\right)^k \sum_{n=0}^\infty \Lambda^n T_{t,n}^{(k)*}(\zeta,\tau),$$
$$0 \le \zeta < \infty, \ \tau \ge 0, \ k = 0,1, \tag{57}$$

$$T_{t,n}^{(k)*} = F_k\left[\left(2n + \frac{\zeta}{\sqrt{k_t^*}}\right)\frac{1}{2\sqrt{\tau}}\right] + \lambda_f F_k\left[\left(2n + 2 + \frac{\zeta}{\sqrt{k_t^*}}\right)\frac{1}{2\sqrt{\tau}}\right],$$
$$n = 0,1,2\ldots, \tag{58}$$

$$T_s^{(k)*}(\zeta,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\left(\frac{\tau}{\tau_s}\right)^k \sum_{n=0}^\infty \Lambda^n T_{s,n}^{(k)*}(\zeta,\tau),$$
$$-1 \le \zeta \le 0, \ \tau \ge 0, \ k = 0,1, \tag{59}$$

$$T_{s,n}^{(k)*}(\zeta,\tau) = F_k\left(\frac{2n-\zeta}{2\sqrt{\tau}}\right) + \lambda_f F_k\left(\frac{2n+2+\zeta}{2\sqrt{\tau}}\right),$$
$$n = 0,1,2\ldots, \tag{60}$$

$$T_f^{(k)*}(\zeta,\tau) = \frac{4\sqrt{\tau}}{(1+\varepsilon_t)(1+\varepsilon_f)}\left(\frac{\tau}{\tau_s}\right)^k \sum_{n=0}^\infty \Lambda^n T_{f,n}^{(k)*}(\zeta,\tau),$$
$$-\infty < \zeta \le -1, \ \tau \ge 0, \ k = 0,1, \tag{61}$$

$$T_{f,n}^{(k)*}(\zeta,\tau) = F_k\left[\left(2n + 1 - \frac{1+\zeta}{\sqrt{k_f^*}}\right)\frac{1}{2\sqrt{\tau}}\right], \quad n = 0,1,2\ldots. \tag{62}$$

Taking the form of function $q^*(\tau)$ (30) and linearity of the boundary-value problem of heat conduction (18)–(29), the dimensionless temperature at distance $|\zeta| < \infty$ from the surface of friction may be presented as the superposition [18,19]:

$$T^*(\zeta,\tau) = [T^{(0)*}(\zeta,\tau) - T^{(1)*}(\zeta,\tau)]H(\tau) + T^{(1)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s), \quad \tau \ge 0. \tag{63}$$

4. Some particular problem solutions

In the case of identical physical properties of a strip and foundation ($K_s = K_f$, $k_s = k_f$) from formulae (16), (44)–(46) it follows that $K_f^* = 1, k_f^* = 1, \varepsilon_f = 1, \lambda_f = 0, \Lambda = 0$. The Eqs. (57)–(63) at $n = 0$ give the solution of the problem of heat generation at braking with uniform retardation for two semi-infinite bodies:

$$T_t^{(k)*}(\zeta,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\left(\frac{\tau}{\tau_s}\right)^k F_k\left(\frac{\zeta}{2\sqrt{k_t^*\tau}}\right), \quad 0 \le \zeta < \infty, \tau \ge 0, k = 0,1, \tag{64}$$

$$T_f^{(k)*}(\zeta,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_t)}\left(\frac{\tau}{\tau_s}\right)^k F_k\left(-\frac{\zeta}{2\sqrt{\tau}}\right), \quad -\infty < \zeta \le 0, \tau \ge 0, k = 0,1. \tag{65}$$

Substituting solutions (64) and (65) into the right-hand side of Eq. (63) at $\zeta = 0$, $0 \le \tau \le \tau_s$, we obtain the Fazekas known formula for the calculation of dimensionless contact temperature [20]:

$$T_t^*(0,\tau) = T_f^*(0,\tau) = 2\sqrt{\frac{k_t^*\tau}{\pi}}\left(1 - \frac{2\tau}{3\tau_s}\right), \quad 0 \le \tau \le \tau_s. \tag{66}$$

At identical material properties of the top semi-space and strips ($K_t = K_s$, $k_t = k_s$) from the formulae (16), (44)–(46) it follows that $K_t^* = 1, k_t^* = 1, \varepsilon_t = 1, \lambda_t = 0, \Lambda = 0$ and we obtain:

$$T_t^{(k)*} = \sqrt{\tau}\left(\frac{\tau}{\tau_s}\right)^k\left[F_k\left(\frac{\zeta}{2\sqrt{\tau}}\right) + \lambda_f F_k\left(\frac{2+\zeta}{2\sqrt{\tau}}\right)\right],$$
$$0 \le \zeta < \infty, \ \tau \ge 0, \ k = 0,1, \tag{67}$$

$$T_s^{(k)*}(\zeta,\tau) = \sqrt{\tau}\left(\frac{\tau}{\tau_s}\right)^k\left[F_k\left(\frac{-\zeta}{2\sqrt{\tau}}\right) + \lambda_f F_k\left(\frac{2+\zeta}{2\sqrt{\tau}}\right)\right],$$
$$-1 \le \zeta \le 0, \ \tau \ge 0, \quad k = 0,1, \tag{68}$$

$$T_f^{(k)*}(\zeta,\tau) = \frac{2\sqrt{\tau}}{(1+\varepsilon_f)}\left(\frac{\tau}{\tau_s}\right)^k F_k\left[\left(1 - \frac{1+\zeta}{\sqrt{k_f^*}}\right)\frac{1}{2\sqrt{\tau}}\right],$$
$$-\infty < \zeta \le -1, \ \tau \ge 0, \ k = 0,1. \tag{69}$$

In the case of identical physical properties of the top semi-space and foundation ($K_t = K_f = K$, $k_t = k_f = k$), the dimensionless temperatures are calculated under formulae (57)–(63), assuming $\varepsilon_t = \varepsilon_f \equiv \varepsilon$, $\lambda_t = \lambda_f = \lambda$, $\Lambda = \lambda^2$, where

$$\lambda = \frac{1-\varepsilon}{1+\varepsilon}, \quad \varepsilon = \frac{K^*}{\sqrt{k^*}}, \quad K^* = \frac{K}{K_s}, \quad k^* = \frac{k}{k_s}. \tag{70}$$

If the materials of all elements are identical ($K_t = K_s = K_f$, $k_t = k_s = k_f$) then in Eqs. (64) and (65) it is necessary to put in addition $k_t^* = 1, \varepsilon_t = 1$ or $k_f^* = 1, \varepsilon_f = 1, \lambda_f = 0$ in Eqs. (67)–(69).

5. Numerical example

The numerical results have been computed for the commercial friction pair ChHMKh cast iron disk (the upper surface) and metal-ceramics FMK-11 frictional element of the patch (strip) on 30KhGSA steel base (foundation), for which [2]:

- ChHMKh: $K_t = 51$ W m^{-1} K^{-1}, $k_t = 14 \times 10^{-6}$ m^2 s^{-1};
- FMK-11: $K_s = 34.3$ W m^{-1} K^{-1}, $k_s = 15.2 \times 10^{-6}$ m^2 s^{-1};
- 30KhGSA: $K_f = 37.2$ W m^{-1} K^{-1}, $k_f = 10.3 \times 10^{-6}$ m^2 s^{-1}.

The friction conditions are: $p_0 = 1$ MPa, $f = 0.7$, $V_0 = 30$ m s^{-1}, $t_s = 3.44$ s. The initial temperature equals $T_{t,s,t}(z,0) = 20$ °C, $|z| < \infty$.

Isolines for the temperature constructed in the coordinate system (z,t) are shown in Fig. 2. The maximal temperature $T_{max} = 740$ °C is reached on a contact surface $z = 0$ at the moment $t = t_{max} = 1.6$ s, which is not much lower than half value of braking time $t_s = 3.44$ s. This result corresponds well with the experimental data $T_{max} = 760$ °C, published in monograph [2, p. 71].

The temperature evolution on a contact surface $z = 0$ for various values of strip thickness d, is shown in Fig. 3. For the fixed value of strip thickness, temperature is increasing rapidly with initialization of braking process, the maximal temperature is reached at the moment which nearly corresponds to half of braking distance (as it was mentioned above for $d = 5$ mm we have $T_{max} = T(0, t_{max}) = 740$ °C, $t_{max} = 1.6$ s). With a time passing by, the temperature decreases to its initial value. The results for the boundary value of strip thickness $d = 0$, were achieved with use of the contact problem solution (63)–(65) for two semi-spaces: a top one made of cast iron and bottom – the steel one. The curve

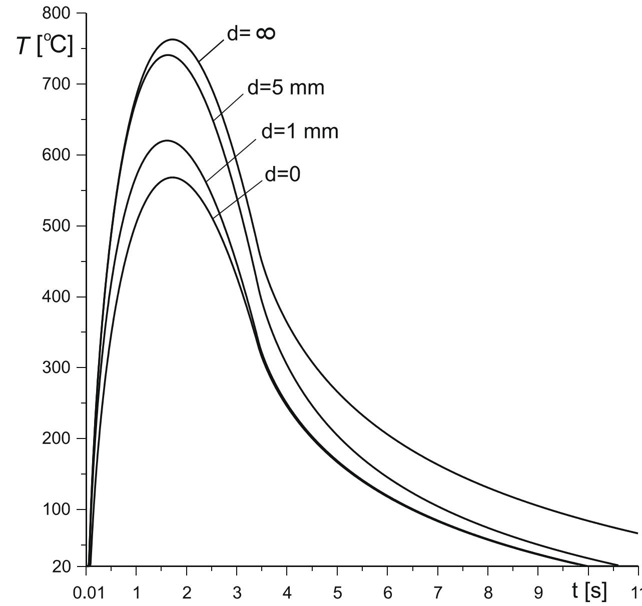

Fig. 3. Evolution of the contact temperature $T(0,t)$ at braking for different values of the strip thickness d.

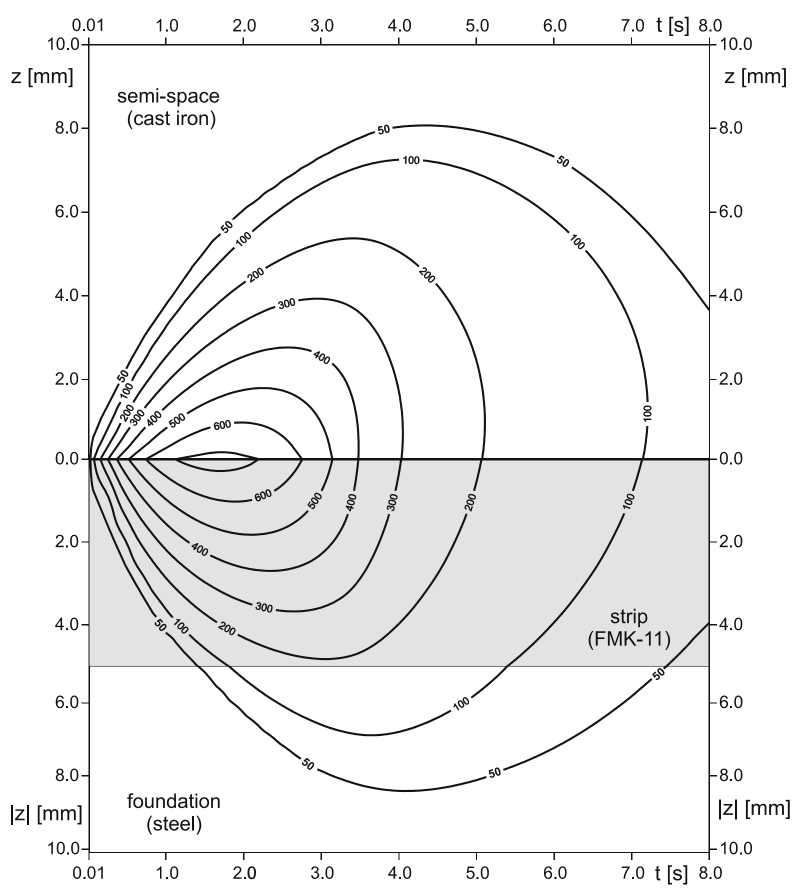

Fig. 2. Isotherms of temperature $T(z,t)$ for the strip thickness $d = 5$ mm.

$d = \infty$ represents data obtained for the top semi-space made of cast iron too, and bottom made of FMK-11 cermet, respectively.

The temperature distribution with distance $|z|$ from contact surface at the moment $t = t_{\max} = 1.6$ s, when temperature reaches maximal value, at the stop moment $t = t_s = 3.44$ and at the moment after being stopped $t = 6$ s, when the cooling of tribosystem being under study takes place, is shown in Fig. 4. It could be noticed then that for $t = t_{\max}$, the temperature decreases with thickness linearly and could be calculated from approximate dependence: $T(z, t_{\max}) = 121.01z + 740.62$, $-d \leqslant z \leqslant 0$.

The effective heat penetration depth, the depth where temperature decreases to 5% of its maximum value on the contact surface, is equal nearly 6.5 mm in both directions from the contact surface.

Dependence of the maximal temperature $T_{\max} = T(0, t_{\max})$ on strip thickness d is shown in Fig. 5. It could be noticed that influence of strip thickness on maximal temperature is significant for interval 0.01 mm $\leqslant d \leqslant 8$ mm. Whereas for values out of this interval, the temperature can be calculated with the use of contact problem solution with frictional heating during braking for two semi-infinite bodies (63)–(65).

With the increase of braking time t_s, the temperature value increase (see Fig. 6). This increase has nonlinear nature and approximately can be described by the following function: $T_{\max} = 4.94t_s^3 - 62.37t_s^2 + 382.46t_s - 37.54, 0.5s \leq t_s \leq 5s$. At the same moment, dependence of time $t_{\max}$, when maximal temperature is reached on braking time has linear nature: $t_{\max} = 0.4624t_s + 0.0353$ for $0.5s \leqslant t_s \leqslant 5$ s.

On the friction surface the dimensionless temperature $T_t^{(k)*}(0, \tau)$ (57), (58) and $T_s^{(k)*}(0, \tau)$ (59), (60) are equal and depend on dimensionless material parameters ε_t and ε_f (44). These parameters are known as "coefficients of thermal activity" [21], where ε_t characterizes thermal activity of the material of the top semi-space relative to the material of the strip, and ε_t – of the substrate to the strip. Dependence of the maximal dimensionless temperature $T^*_{\max} \equiv T^*_t(0, \tau_{\max}) = T^*_s(0, \tau_{\max})$ from parameters ε_t and ε_f is shown in Fig. 7. When the ε_f parameter is fixed, and the ε_t increases from zero to five, the temperature on the contact surface decreases. Regardless of the value of the other parameter ε_f the further increase of ε_t does not change the maximal temperature. When the ε_t is fixed, the increase of ε_f changes the contact temperature slightly. It shall be noticed that the friction pair considered $\varepsilon_t = 1.549$ and $\varepsilon_f = 1.317$, and dimensionless brake time $\tau_s = 2.1$ at $t_s = 3.44$ s and $d = 5$ mm.

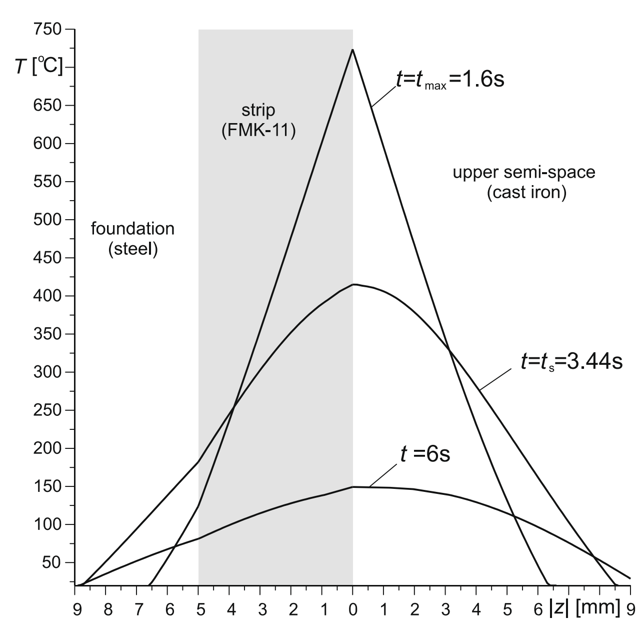

Fig. 4. Dependence of the temperature $T(z,t)$ on the distance $|z|$ from surface of friction for three values of time t at the strip thickness $d = 5$ mm.

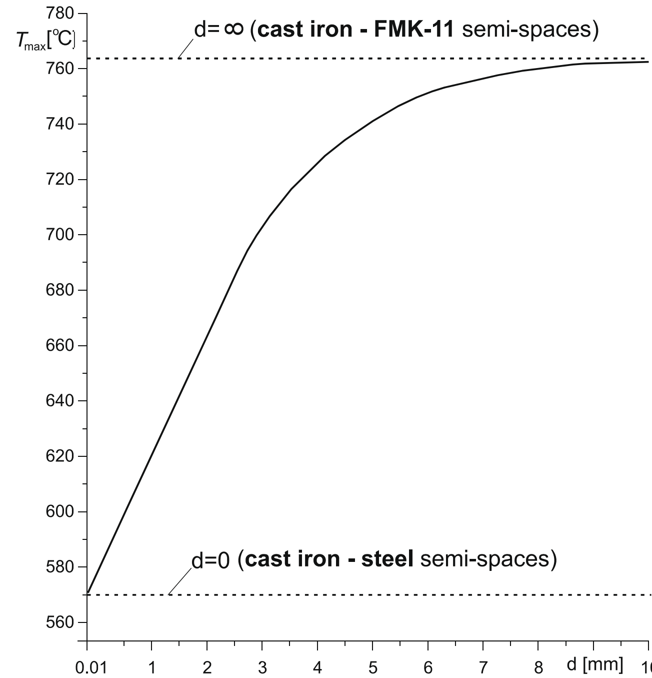

Fig. 5. Dependence of the maximal temperature $T_{\max} = T(0, t_{\max})$ on the strip thickness d.

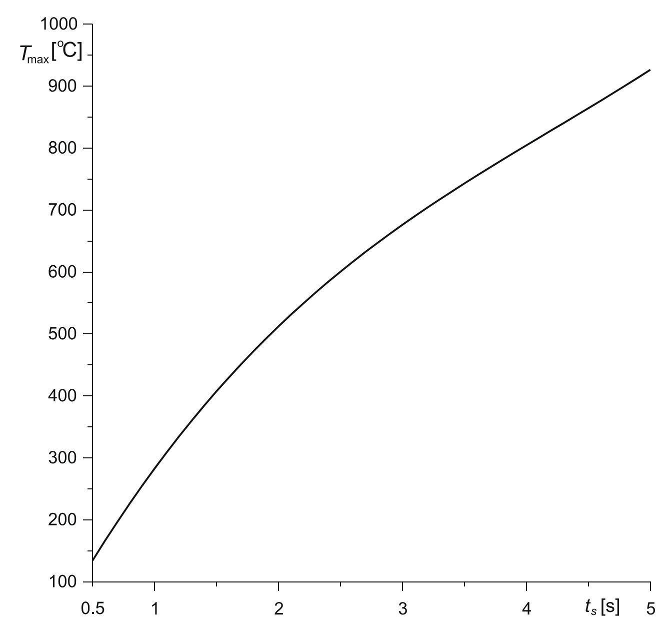

Fig. 6. Dependence of the maximal temperature $T_{\max}$ on the time of braking t_s for the strip thickness $d = 5$ mm.

6. Conclusions

The analytical solution of the transient one-dimensional contact problem with frictional heat generation during braking for the system, which consists of the semi-space and semi-infinite homogeneous foundation with coating, was found. Such solution describes a model of heat generation process during single-braking mode in multi-disk brake. As distinct from other solutions ours

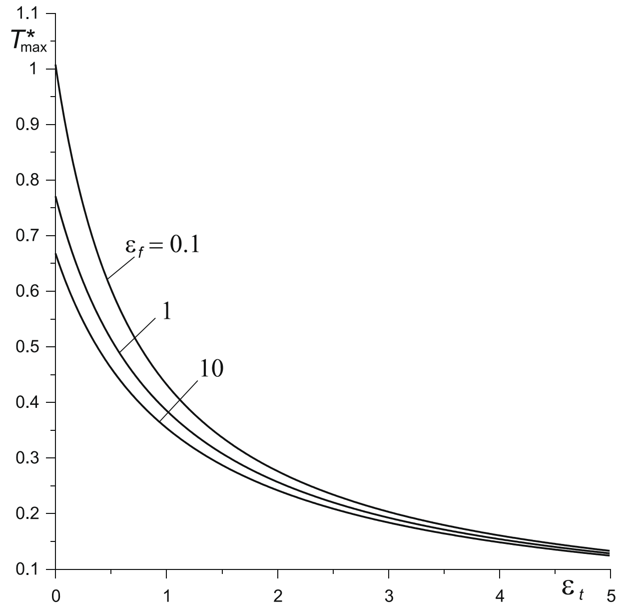

Fig. 7. Dependence of the dimensionless maximal temperature T^*_{max} on the dimensionless parameters ε_t for three values of the dimensionless parameters ε_t at the dimensionless time of braking $\tau_s = 2.1$.

determines the temperature in each element of the tribosystem, both in the heating phase at braking and in the cooling phase, when the brake is stopped. Moreover, the temperature evolution and distribution in relation to thickness of each materials of friction pair: cast iron disc + FMK-11 metal ceramic patch on the steel foundation, were examined. The maximal temperature value $T_{max} = 740\ ^\circ C$, obtained as a result of numerical calculations, corresponds pleasingly with the respective value found in paper [2]. This proves that achieved in the present paper analytical solution of a linear boundary-value problem of heat conduction parabolic type, may be applied in calculations of temperature regimes for defined brake systems even without taking into account complicated dependences of frictional coefficient on temperature.

On the basis of achieved numerical data, the engineering formulas for temperature determination in the arbitrary point inside the strip at t_{max} moment, when the temperature of the contact surface reaches the maximal temperature T_{max}, were proposed. In addition, the respective dependences for T_{max} and t_{max} on braking time t_s, were included.

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